

PROJECTIONS AND CODING IN PULSPROPAGATING NETWORKS

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VIRTUAL EXPERIMENTS

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Introduction

By analogy to a (digital) radio receiver, that correlates thousands of samples in series to get an approximation for a single sound sample, a neuron has thousands of synapses to create a result, working parallel. Comparable to the radio receiver, a theoretical background knowledge is necessary to understand functions in networks of neurons.

Before measuring nerval activities we have to examine and to understand simplest wired, multi-channel and delayed interference circuits. Because it is not possible to construct delay lines, -fields or -spaces in hardware we use our imagination or software simulations - therefore the subtitel 'virtual experiments'.

Note, that interference systems are not cklassical 'neural networks'. Additional, they need physical space and time properties. The results are sometimes opposite, they are mostly not comparable to that obtainable by neural network theories.

In this paper, we introduce pulspropagating networks in the dimension of centimetres. They consist of delaying wires that carry time functions, especially impulses with durations in the milliseconds-range. Impulses moves very slow with speeds in the range of millimeter to meters per second. We follow ways of different pulses and try to examine meeting points. We find conditions for data addressing in chaotic connected networks. We calculate locations of pulse interferences, and find relations between code and space. The conclusion shows relevance for special topics in bio-informatics. All experiments use signal-retentive, delaying wires without attenuation.

Wires in interference schemes are different to electrical wires. An electrical wire implies node properties, where interferencial usage implies low speed and distributed parameters. Bio-neural wires are supposed to be without of attenuation. Wires are not constructed to carry currents or to drive loads. Comparable to nerves they only symbolize the directed flow of time-functions from and to different points with definitive speed.

To demonstrate the different properties, it is not possible to draw circuits in biological form. We are using simplified schemes that allows to understand the main important properties of these nets.

1. Experiment: Simplest Interference Circuit

t_1 and t_2 may indicate the delays of some wiring within a short cutting neuron. A simple, resulting time function $y(t)$ is for example:

$$(I) \quad y(t) = \theta(x(t - \tau_1) + x(t - \tau_2) + bias)$$

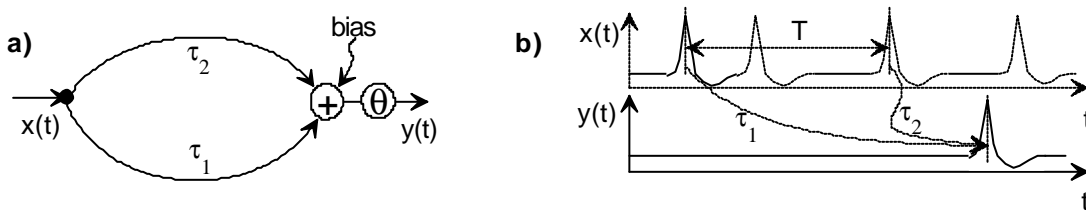


Fig. 1 Interference/superimposition circuit with two delays

I) Supposed, the bias is zero to get any OR-characteristics. Any impulse at the input produces two impulses at the output (not drawn).

II) Next the bias may be negative enough, to get any AND-characteristics (b). Then an output spike only can occur, if the input $x(t)$ consists at minimum of two impulses with a time difference $T = abs(t_1 - t_2)$

III) Last supposed, $x(t)$ may be any spike code having some impulses in distances T and the bias is of AND-type. Interference only occurs in cases, when pulses differ in T . The circuit

acts as a code selector, all pulse groups in T -distance are selectable. Because the delay of any wire degrees with velocity v and grows proportional to its length and thickness d , the geometric properties of the circuit code the selectable T delay group (code generation/detection).

2. Experiment: Slide Location and Time Functions

Two contra-directional wires, each suggested with a *not* neglectable delay, connect with a pulse generator, that produces pulses with a duration $1ms$. The pulses move along the wires with a suggested constant speed of $v = 0.1m/s$.

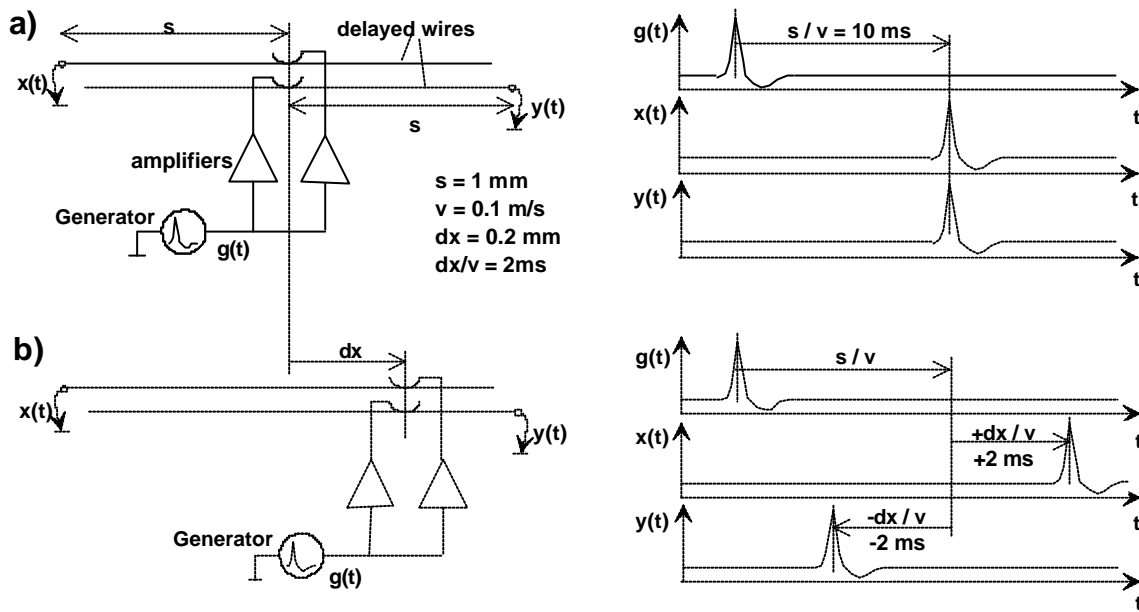


Fig. 2 Timing at the endpoints of delaying wires. a) Slide position neutral; b) slide movement about $+dx$ produces antivalent delay variations of $\pm (dx/v)$ at opposite detectors

We introduce two wires, carrying a pulse with a very low speed v . When the slide stands in the middle position (Fig. 1a), the resulting time functions $x(t)$ and $y(t)$ have the same appearance. Moving the slide by a distance dx to the right side, the pulses walk in different directions with the amount $\pm(dx/v)$. Instead of the slide we can use a row of pulse-generating elements, for example neurons.

3. Experiment: Timing Influences the Location of Exciting Neurons

Let us consider the opposite case: We use two generators at the end-points of contra directional going wires. The pulse sources may be synchronised with a delay of $4ms$. Thus, the output impulses $g(t)$ arrives with $T = 4ms$ delay.:

$$(2) \quad y(t) = x(t-T); \quad g(t) = x(t-s/v) + y(t-s/v) = x(t-s/v) + x(t-s/v-T)$$

When we move the slide out of the centre position, for example to the right side, the delay difference between both pulses increases. When the slide moves to the left side, we find a position, where both partial impulses merge to one pulse with a double amplitude.

$$(3) \quad g'(t) = x(t-(s/v-dx/v)) + y(t-(s/v+dx/v))$$

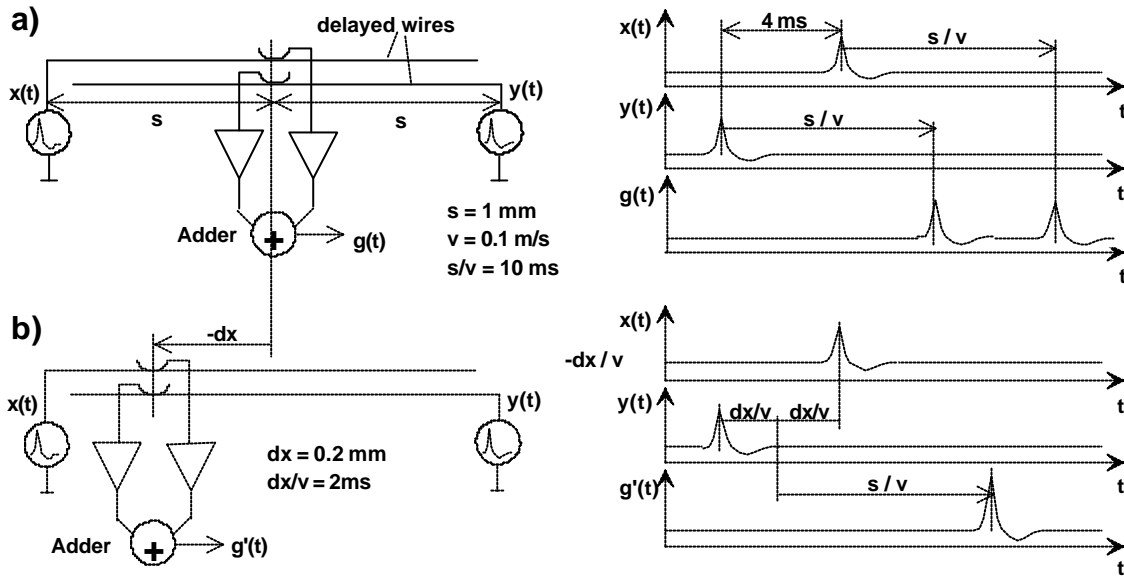


Fig. 3 Search for the interference point, inputs are $x(t)$ and $y(t)$ from Fig.1.
a) Slide in centre position produces two pulses; b) We have to move the slide in the opposite direction as in Fig. 1 to find interference

We call this point the location of interference. If we use neurons with a threshold of 1.5 (the height of pulses may be 1), and we exchange the slide mechanism through a row of such neurons, only that neuron fires, that lays in the interference location. All other neurons keep still. Notice, that the position of an excited neuron varies with varying time functions. Easier we reach the same result using *neurons with a multiplying operator*. Then also that neuron fires only, that lays in the position of interference.

4. Experiment: **Basic Interference Circuit, 1-dimensional**

We use the resulting time functions of Fig. 1 to generate inputs at opposite vertices. Moving the slide (Fig. 2) we see two different pulses.

Only in one position ($-dx$) we get a large excitement, with an increased amplitude. Notice, that we moved the slide to the *opposite direction* as in Fig. 1, to find the position of interference. (Self-) Interference occurs, when the *sum of partial delays* t_i on all possible ways with n parts is *constant*:

$$(4) \quad \sum_{i=1}^n \tau_i = \sum_{j=1}^n \tau_j = \dots = \sum_{k=1}^n \tau_k = \text{const. (interference condition)}$$

If the slide of the generator is moved away from the interference case, a single pulse at the detector disappears. We have to correct the position of the detectors slide, to reach a single pulse at the output. In case of interference is $dx = dx'$, thus follows $g'(t) = 2 g(t-(2s+s')/v)$. The importance of interference grows, as higher the complexity of the interference circuit is. If we use some hundred instead of two connecting wires, and if the neuron has an very low threshold, a neuron becomes very time- or location- sensitive. If we suppose a multiplication-operator instead of an additive operator for the detector, we see that each neuron produces an output in the case of interference only.

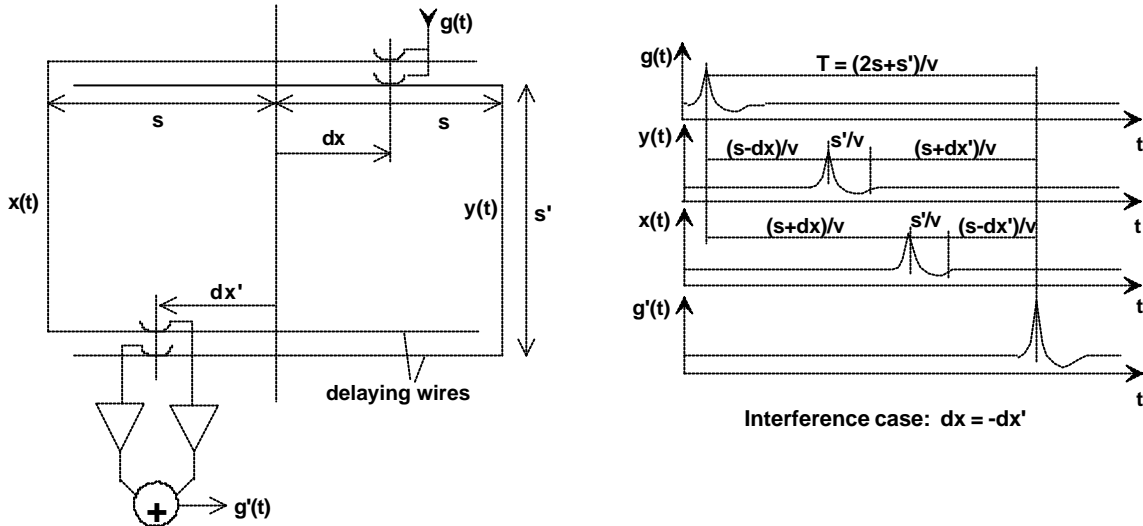


Fig. 4 Basic, one-dimensional interference circuit. Interference maps the receiving field topographical mirrored to the sending field

5. Experiment: The Cover Codes the Behaviour

Now we vary the geometrical properties between wires. Forming the bottom field in a circular way a), the properties stay unchanged. The image size varies, if the speed between the fields changes. A realization b) of an easy interference circuit needs non contra-directional wiring.

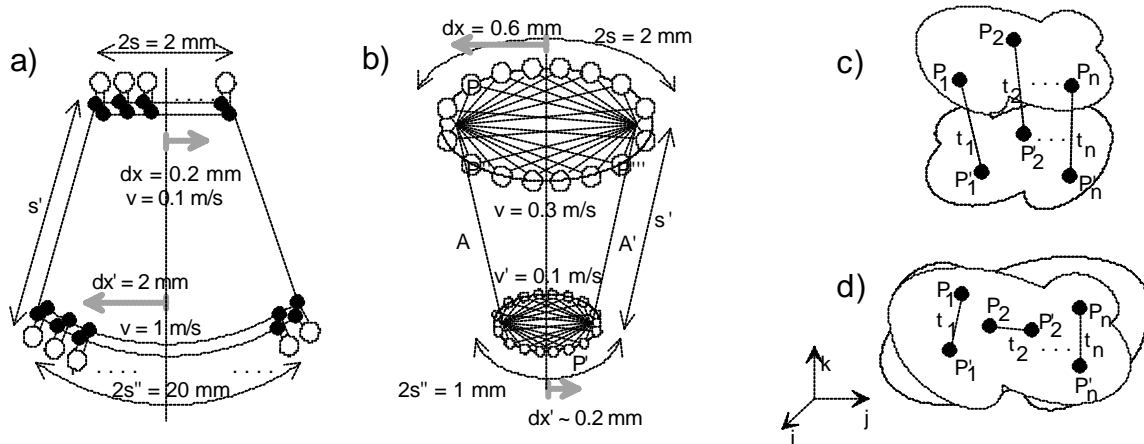


Fig. 5 Other forms of atomic interference circuits. a) A projection occurs with different speeds. b) Projective circuit with discrete neurons. Generalised interference circuits c) and d)

If a neuron fires in position P , both wave parts move through the circuit b), the left one through A , the right through A' . Supposed, bottom detectors have a threshold between 1 and 2, and pulse height is 1. Interference is the only condition, to excite a neuron. Thus the image occurs mirrored to the firing neuron P in P' . The accurate form, parameters and locations of the connecting wires A, A' together with the location of the neurons decide the correct function of the circuit. The nature of the circuits is unidirectional. If the connecting wires are uni-directional, we can arrange generating and detecting neurons in a free manner at each side. In opposite to logic-circuits it is *not* possible expecting the same function (with the same interference location) to shift any neuron to another

place. It seems important, to notice the construction rules of projective circuits. If the connecting wires have defined source- and destination co-ordinates in relation to the corpus of neurons, it is necessary only, that the dendrites connect both. In opposite to conventional electric circuits we need only a pure cable plan. **Law: The function of one neuron is coded through its relative location.** Interference conditions demand equal delays on all partial paths. In multi- channel approaches this fact may be confusing. A way to reach sharp interference projections is, to lay all neurons on ellipsoidal curves, bringing the connecting wires into the focal points *b*). To construct a shell that fulfils these condition, compare the architecture of the cerebellum. Penfields 'Homunculus' may be the first idea while inspecting possible layouts.

Virtual projections (dashed lines) get interferences in different ways: if the source is *P'*, at the other side in *P''*. If the source is on the same side in *P*, the excitement arrives in *P'''*. We can generalise interference circuits by sorting neurons, that are connected (direct or indirect) to source- or drain-points of one field *c*). It seems possible, that fields merge in a very closed way *d*). Notice, that in opposite to Kohonen's feature maps interference images are *mirrored* on principle.

6. Experiment: Addition and Multiplication of Time Functions

We use two easy Gauss-impulse series *imp5(t)* and *imp6(t)* to demonstrate some possibilities of superposition of time-functions.

$$(5) \text{imp5}(t) = \sum_{i=1}^{\infty} e^{-(t-5i)^2}; \quad \text{imp6}(t) = \sum_{i=1}^{\infty} e^{-(t-6i)^2}$$

The functions produce impulses every 5 or 6 units of time. Comparable to a neuron, an arithmetical operator should calculate a normalised sum or a product *u(t)* stepwise. Step by step we combine the current values of incoming time functions to calculate the resulting time function. In different cases appear difficult time functions for product and (normalised) sum. Using a high threshold, the results are nearly comparable; see threshold a). With other thresholds c) we get different results between addition and multiplication. It is interesting, to see weak intermediate products and sums b).

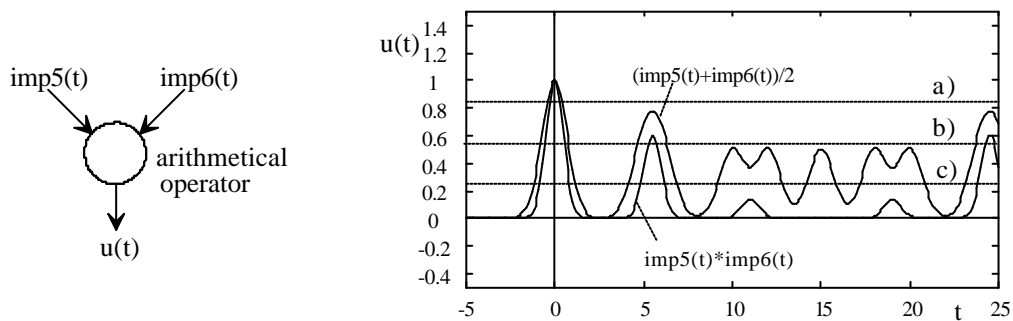


Fig. 6 Comparison of added and multiplied time functions. Some thresholds show comparable results a) or b). Other thresholds c) produce very different time functions

Notice, that the form of an impulse is *unchanged* with (normalised) addition-operators. In opposite, a (normalised) multiplication operator *decreases* (mathematically) *the width* of a (self- multiplied) impulse (see pulses for *t = 0*). For example: Supposed, we use a multiplying gate with 1000 inputs. In case of interference the pulse- width at the output (Gauss) decreases to 6.3%. Interference of many multiplying inputs enhances the selectivity of a gate dramatically. For comparison: an average neuron has 7640 synapses.

7. Experiment: Zooming

We vary the speeds of different wires of an interference circuit. The speed v of the generator circuit G now is different to v' of the detector circuit D . On both sides we can follow the way of an impulse through space and time. Thus speed is the quotient between delta x and delta t , the gradient dx/dt changes with the speed modification.

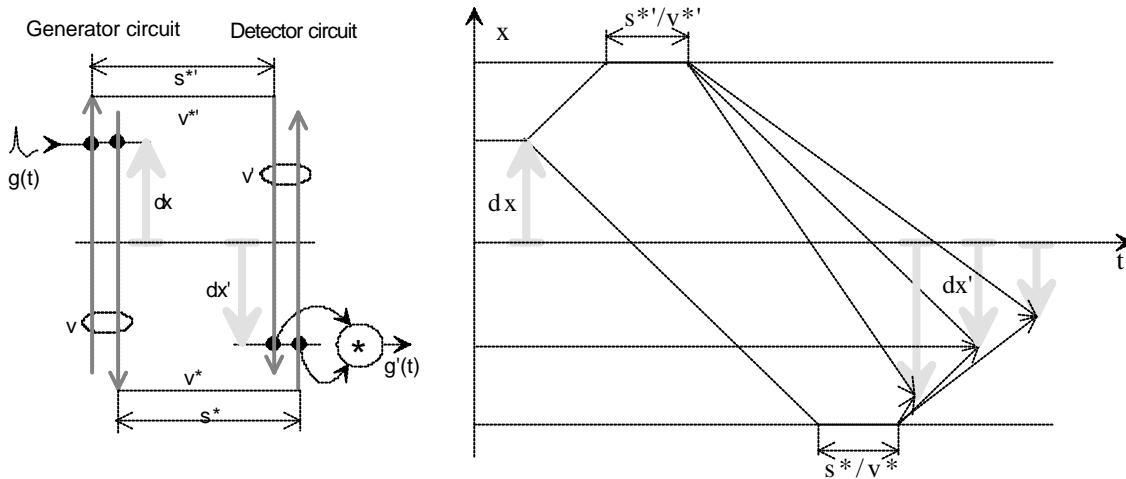


Fig. 7 Different speeds between generator and detector zoom the projection

To get equal delays at all possible ways, the ratios dx/v and dx'/v' have to be equal

$$(6) T = dx/v = dx'/v' \quad (\text{interference condition})$$

thus, the ratio of image sizes between detector (') and generator circuit is nearly

$$(7) dx/dx' = v'/v \quad (\text{size ratio for zoom})$$

the sizes are indirect proportional to the corresponding speeds. For example: The speed of the detector circuit is n times slower, then that of the generator. Then the detector produces a n times higher image size, as the generator.

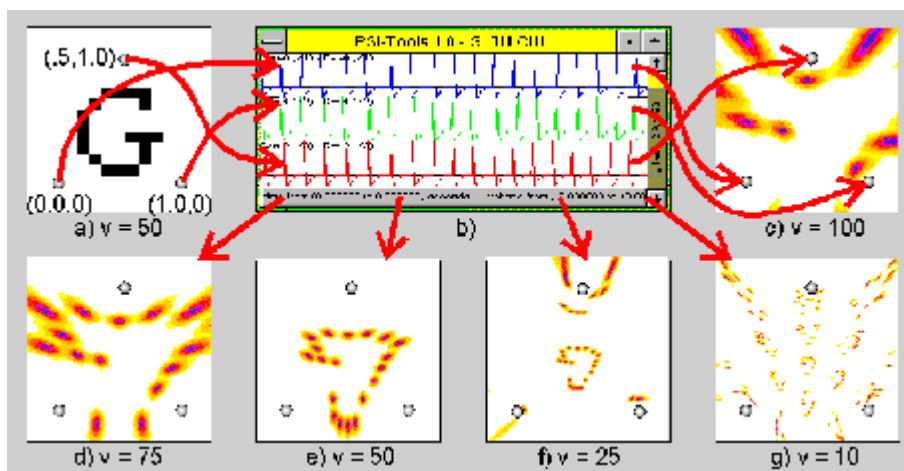


Fig. 8 Zooming interference projections in 2D using HIT and PSI-Tools. a) Generator space with speed $v=50$, black pixels fire one for one; b) channel data; c)...g) projections with different speeds v

8. Experiment: Moving

Maybe, the generator G and detector D circuits speed stays unchanged. Instead we differ the delay (T) or the speed ratio between the wire $v^{*'}$ and the wire v^* connecting detector with generator circuit. To get equal delays on all possible ways, the delay of the upper axon (wire) is equal to that of the bottom axon.

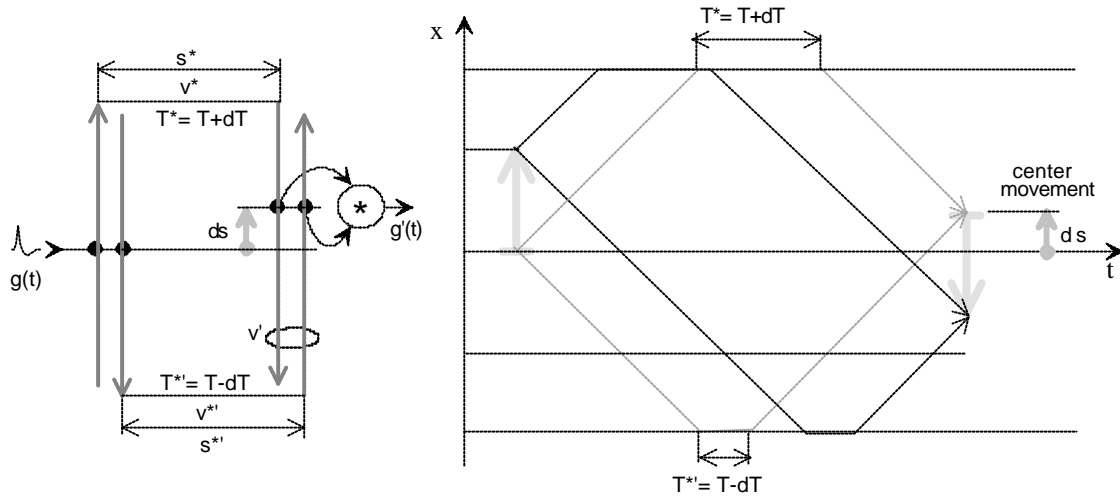


Fig. 9 Different delays between the transmission wires shift any interference location (hence the image centre) to the side of the higher delay

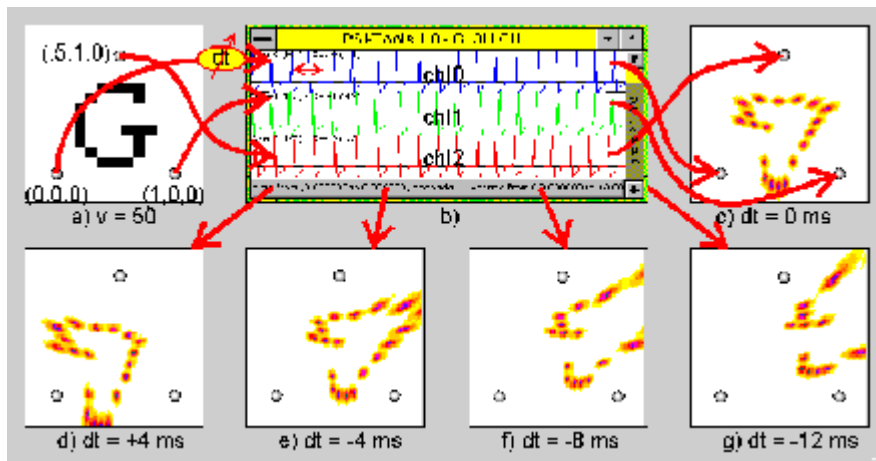


Fig. 10 Moving interference projections in 2D-space using HIT and PSI-Tools.
 a) Synthetic generator space; b) Channel data;
 c)...g) Reconstruction's with delayed channel 0

If the delay of both connecting wires changes by $\pm dT$, the image centre moves.

$$(8) \quad ds = v' dT \quad (\text{image out of centre})$$

That means, a higher delay at the upper side produces a centre movement to the upper side! The size ratio stays constant. But, the image itself moves out of the centre position. It would be possible to use this circuit for delay compensations, for angle measurement or for the measurement of small length variations, coupled with delay modifications.

9. Experiment: Delay Mask of a Location

We use a virtual homogeneous connected medium. At the location P a single pulse starts. A pulse-wave expands concentric to P . Different located electrodes supposed, the pulse-wave reaches at different times the different electrodes. The time difference is proportional to the distance between P and the electrode.

Each location P is concerned by a special set of relative radiuses or delays. Thus, the relative delays to definitive electrodes we call the **mask of P** . We need the mask to locate the sending location P in all cases, we have to interpret channel data of a space. Notice, that a point need at minimum a $n+1$ -dimensional mask ($n+1$ channels), where n is the dimension of space.

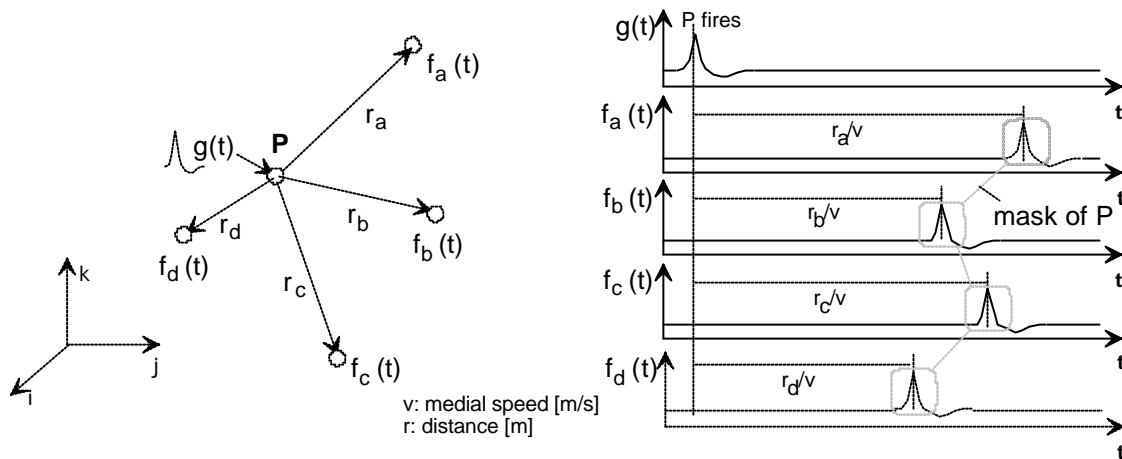


Fig. 11 When P pulses, at the sensors $f_i(t)$ the time-function appears delayed. A neuron at point P sends a typical 'mask', which codes the own location

In relation to the electrode locations the signals appear different delayed. If P is geometrically shifted, the delay-mask of P varies. Thus *each location has its own mask*, related to fixed electrode positions. If we get a data stream about an unknown space from electrodes, under positive circumstances with this knowledge the reconstruction of firing points is possible.

10. Experiment: Phased Array Compensation

May be, the task is, to divide one acoustic source from others, or to observe a source located in the middle of other, noisy sources. In case of an infinite distant source we know phase compensation techniques, comparable to Yagi- antennas or Phased-Array antennas.

In case of a not infinitive distance we only can use the mask of the source to compensate the channel data. We shift the complete data set of all channels j with the actual delay distance T_j between each source and sensor element (electrode, microphone) left at the time axis, in terms:

$$(9) \quad t_{compensated} = t_{measured} - T_j$$

with T_j is the delay between the source and the sensor element with index j , see figure above.

To get better signals of the source point P we have to multiply or to add all compensated channel data in a manner like neurons or like correlative systems do. Practical experiments show, that (in very dependence of the arrangement) the signal to noise ratio grows proportional to the maximum distance between sensors, or to the number of sensors, if the detected wavelength is small compared with the dimensions of the arrangement.

11. Experiment: Reconstruction of Channel Data

Supposed, the suppression period between impulses is large enough. Trying to find out, if a neuron P has fired (reconstruction of a generator field) or if a neuron has an interference (projection into a receiving field) we have to solve a comparable task.

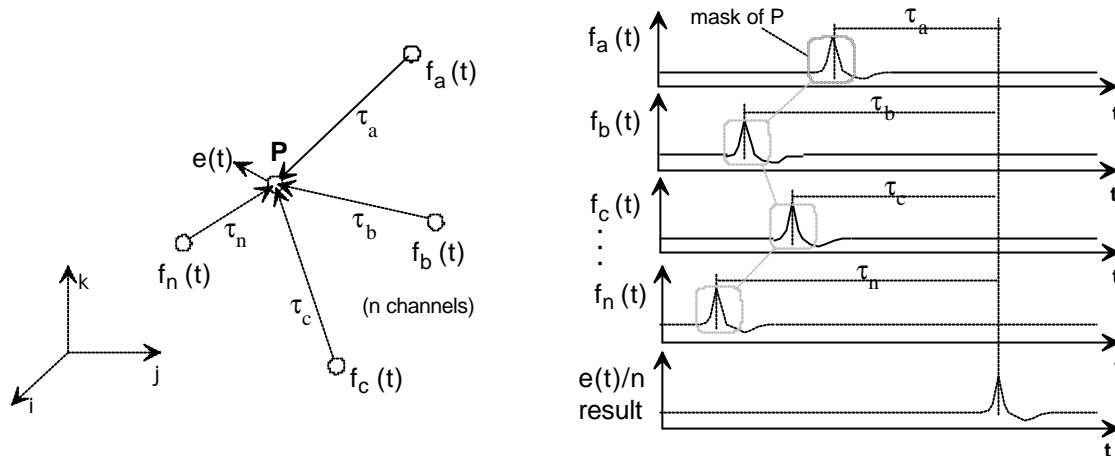


Fig. 12 Reconstruction/calculation of interference images. Only in case of interference a single result impulse $e(t)$ appears.

The mask of the point searching for has to move over the channel data set. The detection of the mask of the point we look for gives an higher interference value at the output $e(t)$. Integration over the total channel stores a one times detected interference at the point.

We assume, the used channel data $f_i(t)$ are valid at unchanged electrode positions.

Like the neuron, sitting at the considered place, we use the same algorithm to combine the incoming data. For example, we can add or multiply the data set along the mask, we can use different neurons types.

If we meet a group of impulses, having the searched mask, a very high interference value occurs. We have to integrate over all steps through the channel data to see, if this point was excited. Attention: If an addition purely is used as neural model, the resulting image stays gray. In this case all neurons get the same amount of interference.

We have to execute this integration over the hole channel for all interesting points of the interesting space. Notice, that each point (neuron) has a different position and thus a different mask! When we finished all channel integrations for all points we can nearly imagine the computational power of a biological system.

12. Experiment: Over-Conditioning Interference System

The construction of a high order transmission over some more channels sacrifice some properties of interference systems. To transmit an interference of an n dimensional space, we need $n+1$ channels. In the example, a two-dimensional field needs at minimum three channels. If we use more channels, the interference system is over definitive. The consequence is, that not all possible detector geometry's interfere in such an over-conditioning system. An example shows this problem.

A point P may lay in a two-dimensional generator field. The point may send a pulse at the time

$t = 0$ The corresponding timing diagram shows, that the impulses reach the corners at different times.

Now we will take the same pulse diagram, and send this through a detecting field. The maximum interference occurs at the point P' (if such a point exists) which has the same delay over all possible ways PP' . The connecting wires between the generating and detecting fields supposed to have the same delay. The absolute delay of the wiring is not important.

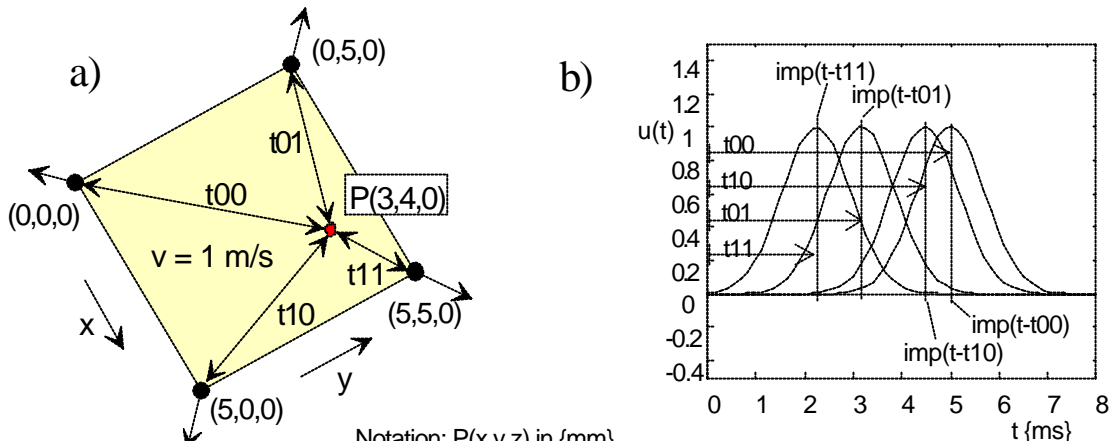


Fig. 13 Pulse wave expansion in a homogenous propagating field without attenuation. Point P pulses at $t = 0$. a) Field; b) Timing at the corners. Point P pulses at $t = 0$

$$(10) \sum_{i=1}^n \tau_i = const \quad (\text{interference condition})$$

Thus we can conclude, that relative delays have to be conjugated between both fields.

Homogenous space supposed, we calculate delays with

$$(11) t_{ij} = r_{ij}/v = \frac{1}{v} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}, \quad (\text{delay})$$

$$(12) \text{ where } t_{00} = 5.0000 \text{ ms}; t_{01} = 3.1622 \text{ ms}; t_{10} = 4.4721 \text{ ms}; t_{11} = 2.2361 \text{ ms}.$$

The interference condition in case of equal wiring delay between generator and detector is:

$$(13) t_{00} + t'_{00} = t_{01} + t'_{01} = t_{10} + t'_{10} = t_{11} + t'_{11} = const.$$

We can check the equation for the source $P=(3,4)$ and a supposed destination at $P' = (2,1)$:

$$(14) 7.236 \text{ ms} \Leftrightarrow 7.634 \text{ ms} \Leftrightarrow 7.634 \text{ ms} \Leftrightarrow 7.236 \text{ ms}.$$

The differences show, that $P' = (2,1)$ is not an exact location for interference! In case of interference all total delays at all paths must be the same! Comparable to an optical lens system, the example shows, that the used geometry is not able to give exact interference at all places! Only the locations near the centre of the generator and detector fields are nearly good in interference.

Over-conditioning supposed, the geometry of two planar areas for detector and generator *can mostly not* (exactly) fulfil the requirement of equal delay paths! The mismatch grows, as higher is the number of channels. We need special composed spherical layers, to find some exact solutions for over-conditioning systems.

The detector field may consist of neurons, all having four inputs connected to the corners of the field. Also it is possible, to understand such a field in a more discrete version with a projection-trajectory, comparable to that shown in its two-channel version above.

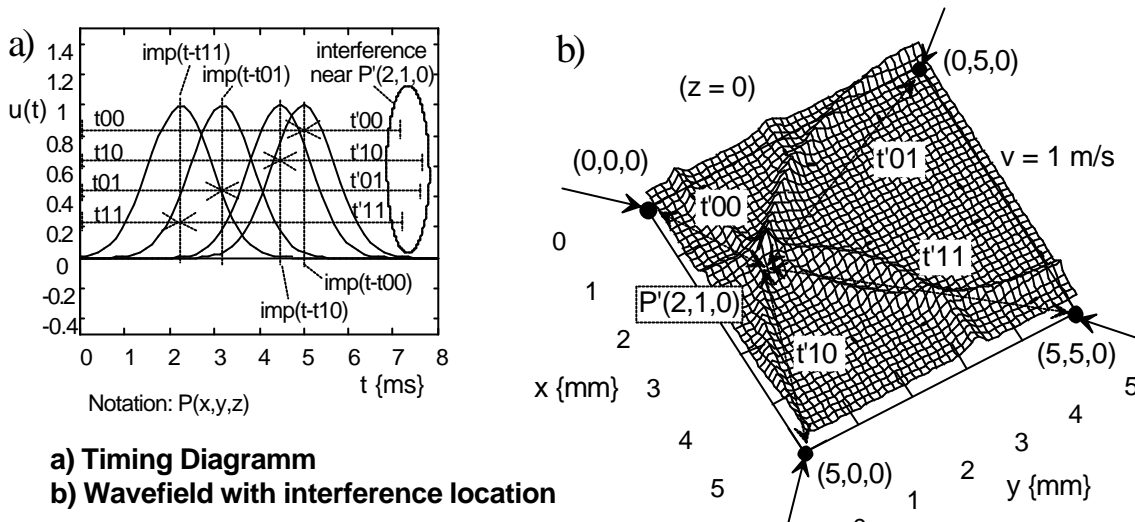


Fig. 14 A possible detector wave field in the minute, when interference occurs near $P'(2,1,0)$. The interference location appears over-conditioned

It seems important, to see also some smaller interference locations, where partial interferences occurred. Partial interferences are reduced, if we use a large amount of channels. To reduce this smaller interferences in relation to the large (real) interference, also we can use a high threshold function, for example by the usage of multiplying detectors (neurons).

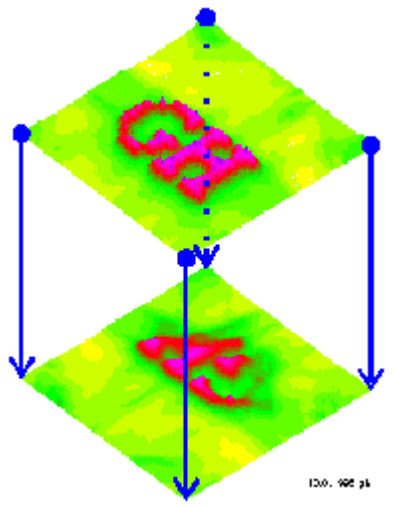


Fig. 15 Comparison of reconstruction and projection of the same channel data. The reconstruction excitement is equal to the generators. Good interference projections occur in the central region.

13. Experiment: Cross Interference, Maps of Serial Code

The next experiment needs some physical fantasy. The problem is now, to calculate the interference density in relation to the co-ordinates of points, instead of a time-dependency.

To simplify, sensorial excitement is supposed periodically [1]. Sensorial amplitude A is coded with a fire rate f

$$(15) f(A) = k A$$

k may be a proportionality constant or a proportionality function.

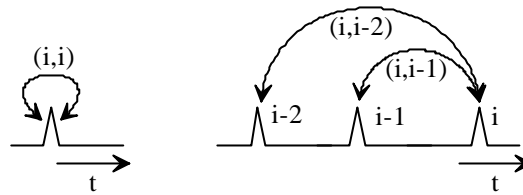


Fig. 16 Self-interference a) and b) cross-interference between three impulses

The experimental arrangement is simple: we need two contra directional wires, both connected symmetrical to one generating point G . Any time function in G produces locations of interference, and other locations, where never an interference occurs. Fig. b) shows an intensity plot for a periodically input sequence. We find, that each impulse i meets itself in the centre position (i,i) only. Any impulse i meets its follower $i+1$ in the position $(i, i+1)$ and so on. We have to note, that all neurons, laying in the space between two intensity maxims, have never the chance, to get any impulse excitement at the same time. They not allowed to receive data of this kind or generated by the source G . They are not 'addressable' within this circuit. Notice, steady impulse series supposed, that the different interferences occur exactly at the same time.

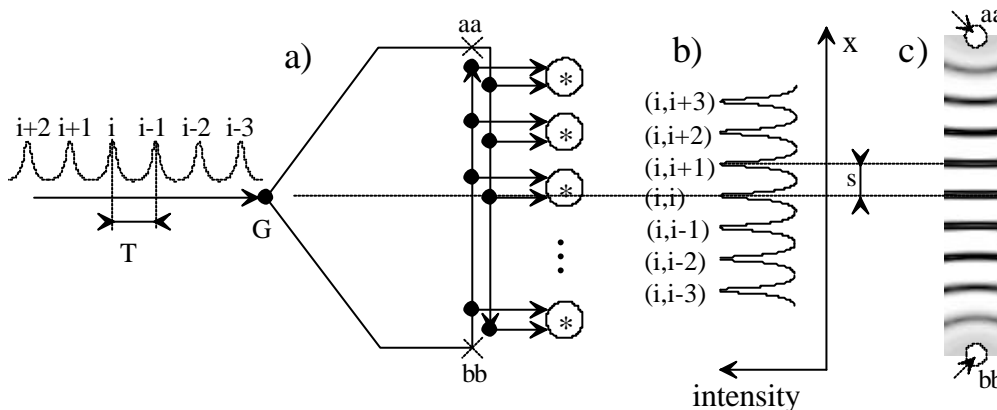


Fig. 17 Experimental approach to diffraction/coherence properties a) circuit, b) intensity diagram for long sequences, c) simulated, hyperbolic 2d-interference with 10 impulses (PSI-Tools)

Between the input pulse frequency $f = 1/T$ and the spacing between maxims s we find the ratio

$$(16) s = vT/2 = v/(2f)$$

Supposed instead of two wires we use a dense layer of neurons c). Each neuron is connected direct or indirect to the source points aa and bb . The information from source points to neurons may use the shortest way. If points aa and bb are the focal points, we get a set of hyperbolas. A hyperbola is the location of all points, that have the same delay difference to the focal points.

To simulate such systems we developed 'Parallel and Serial Interference Tools' (PSI-Tools). Inputs are the channel data and the locations of source points, output is the interference image c).

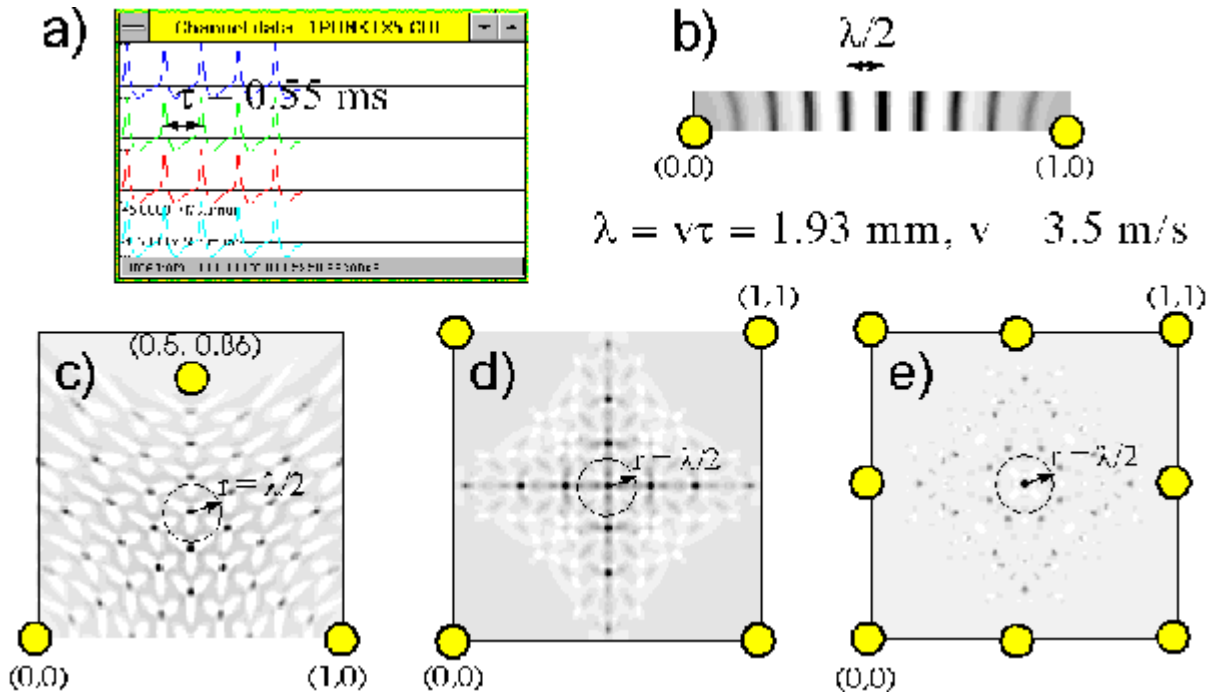


Fig. 18 Neural interference pattern of different channel arrangements in 2-dim. space. Electrode positions are marked. As higher is the channel number, as higher is the pick of the central interference: b) 2-chl.; c) 3-chl.; d) 4-chl. with channel data in a); e) 8-chl. arrangement (PSI-Tools)

14. Experiment: Two Channel Calculation Example

To calculate the intensity value $u(x, y, z, t)$ of a (multiplying) neuron, it seems necessary to use special tools, for example PSI-Tools or in the special *case of two inputs* (Huygens) the transformation

$$(17) \quad u(x, y, z, t) = \frac{1}{c_m^2 T} \int_0^T f(t - \tau_{aa}) f(t - \tau_{bb}) dt \quad (\text{two channel multiplied pattern})$$

The resulting map shows hyperbolic curves for the maximum. In homogenous space the delays are proportional to the distances. As proportionality constant acts the speed v .

$$(18) \quad \tau_{aa} = \frac{1}{v} \sqrt{(x - x_{aa})^2 + (y - y_{aa})^2 + (z - z_{aa})^2} \quad (\text{delay to } aa)$$

$$(19) \quad \tau_{bb} = \frac{1}{v} \sqrt{(x - x_{bb})^2 + (y - y_{bb})^2 + (z - z_{bb})^2} \quad (\text{delay to } bb)$$

T has the rule of a normalisation constant and t_{aa} and t_{bb} are delay distances to the source locations, for example to the channel source points aa or bb . Notice, the delays are not complementary!

x, y, z are the co-ordinates of the current point to be calculated. In the special case of two multiplied time functions we can compare interference with correlation. In the case of cross interference examination of two channels $u(x, y, z, t)$ seems to appear as a special form of an auto-correlation constant.

Notice, interference formulas are not convolutions from the physical point of view. While data flow is opposite or nearly opposite through space, all data flow forward (not opposite) in time. But from

a mathematical point of view it is possible to show, that we can calculate special interference systems (as the discussed) in an effective manner, but only formal as convolution. To calculate interference as convolution, we have to translate the incoming time-functions into location-functions. In case of points getting inputs from opposite directions, in analogy we can convolute the location-functions to get the resulting interference map.

The resulting function $u(x, y, z, t)$ could have the unit of an energy. Because we have to declare interference for different neural operators (addition, multiplication...) and for different channel numbers as synaptic weights, it seems not even important, to analyse the measuring unit(s). Introducing a constant c it seems possible, to solve this physical problem satisfactory for cases of practical relevance. In the case of two multiplied channels c_{m2} simple is 1 Volt. The index $m2$ may stand for *multiplication with two* time functions. Further mathematical possibilities and problems are discussed in [2].

15. Experiment: Phantom Excitement

We have to differentiate between real and phantom excitement. In our speech, points are *real excited*, if they have a representation in the result-space, that is a projection of the source-space.

Examining two pulse groups of two points laying in near distance to each other, we find a second form of excitement. Because the detector field does not know, which impulses came from which source point, a mix of pulses from the first and the second impulse group produces new interference locations. To get fixed interference location, between total channel number n and dimension of the field d we find the equation

$$(20) \quad n \geq d+1. \quad (\text{interference fixing})$$

We call this phenomenon 'phantom excitement'. If the actual channel number is equal to the dimension of space, every phantom excitement has a real location - exists real. If the channel number is higher, every excitement is over conditioned, and it is not clear, if any phantom excitement has real location(s) with high interference values.

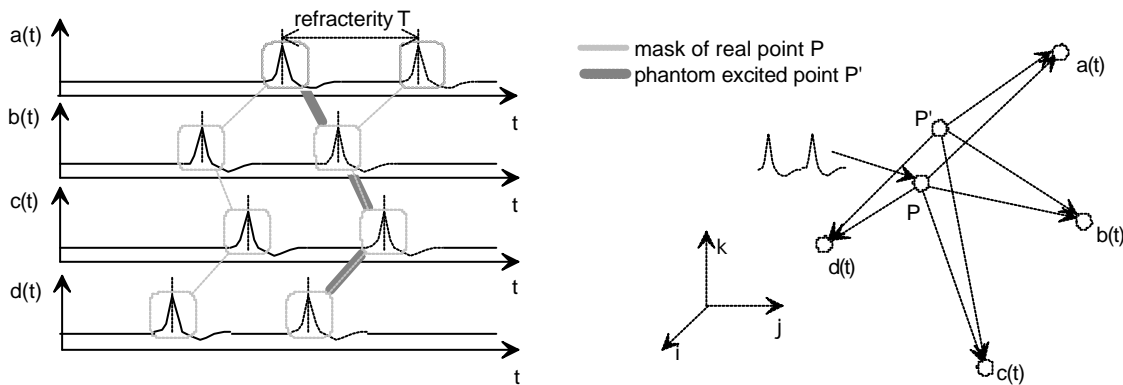


Fig. 19 Small intervals between impulses T supposed, phantom excitement P' appears, where P' can have real or virtual locations

The possibility of *phantom excitement* is as higher, as less channels are used, and as closer is the packing density of impulses. In medical speech: as smaller is the pausing period of transmitting nerves, as more phantom excitement occurs in the detector space. To avoid phantom excitement the delay-distance (interval T) between following pulses has to be greater the time, a pulse need, to move with a speed v over the receiving field of a (maximum) length d with $T \geq d/v$. Thus interval T

and widths d of the detecting field are dependent properties of a pulse-propagating system:

$$(21) \quad T = \frac{1}{f} = \begin{cases} > d/v : & \text{self interference} & (1) \\ \leq d/v : & \text{cross \& self interference} & (2) \end{cases}$$

Remark, that simple experiments with impulses walking slowly through space can help to understand the nature: while case (1) allows topographical mapping, case (2) shows, that sensorial information in multi-channel systems never can be stored alone without self interference. Every sensoric informations in form of cross interference pattern have intrinsic components of self interference, of what we call 'imagination'. This can be seen as a reason in the case someone has (in strange contrast to computers) no imagination without emotion.

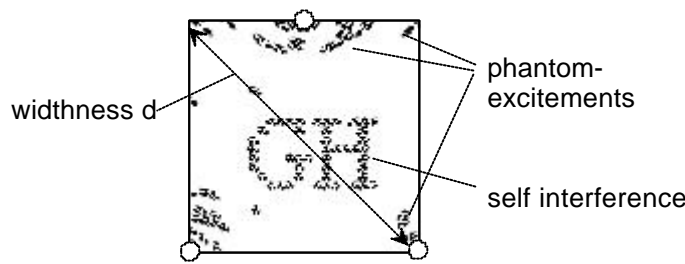


Fig. 20 Phantom excitement reasoned by dense channel data (3-channel-reconstruction). Each point has fired only once

At the other hand, every sensor pulses more than one times. Thus, every self interference carries cross interference pattern, else it is out of the considered field. But: in interference systems with higher channel count (over-conditioned systems) we find a strong decrease of cross interference pattern in simulations. Fire of different neurons can produce the same effect, when intervals are to small.

16. Experiment: Conjunctive Images

We have seen above, that interference is independent of absolute time. Only the (differential, relative) timing between channels is relevant to create interference images and cross interference pattern.

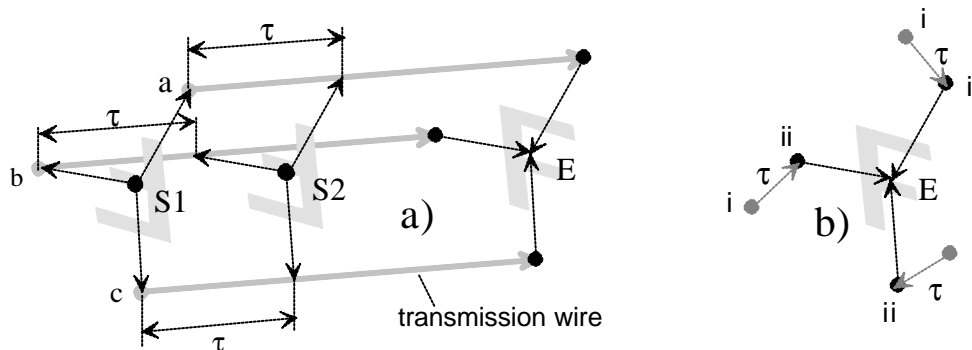


Fig. 21 Conjunctive images: a) transversal, b) radial form. Interference is independent of absolute time t . Detector field E cannot distinguish between $S1$ and $S2$ excitement

We use two generating spaces $S1, S2$ to examine this fact. Both generators may connect at the same wires a, b, c . The spacing between all source points of $S1, S2$ may be t . The question is, if it is possible, to decide, if a received interference in E is reasoned by $S1$ or by $S2$? In this constellation it is not. Thus, the receiver E gets all interferences together coming from $S1$ and $S2$, and any phantom interferences between them together! The same effect can be created by a radial arrangement b). We can add any wires (i-ii) to the source points of the interference field without any influence on the image E .

To prove this hypotheses, we use an experiment [3]. Two virtual generators 'g' and 'h' produce independent channel data streams 'g.chl' and 'h.chl'. Large intervals supposed, both channel data streams are to add or to append: the resulting reconstruction (bitmap 'gh') appears unchanged. More: it is not possible to say, which excitement in the detector space comes from which generator space ('g' or 'h')!

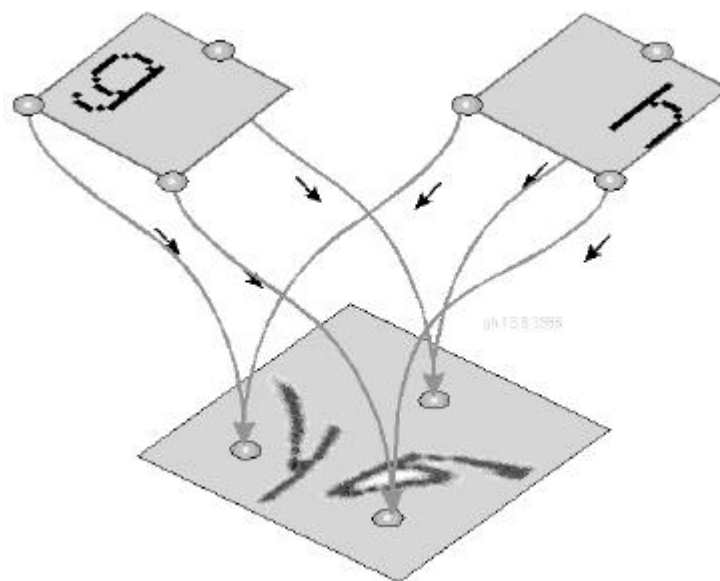


Fig. 22 Projection from two separate generator spaces into a receiving field. It is not possible to decide, which interference comes from which generator field, model for phantom pain (HIT in PSI-Tools)

17. Experiment: Burst Generation & -Detection, Location Sensitivity, Neighbourhood Inhibition. Neural Basic Functions

To understand, what task can fulfil a short-connected neuron, we design a circuit with *different delays on all input wires*. The input weights may be equal.

The input may be a single impulse with the time function $x(t)$. After a time t_i, t_j, t_k this impulse appears at the cell body. In dependence what neuron type we prefer, and which bias we choose, we get different outputs.

Fig. x) shows the case of a very slow threshold (OR-type). We find, that a single input impulse produces several output impulses, leaving the neuron with inherent delays of the neural input wiring themselves [1].

$$(22) \quad y(t) = \theta(x(t - \tau_i) + x(t - \tau_j) + x(t - \tau_k) + \dots + x(t - \tau_m)) + bia:$$

The function q may be a sigmoid- like, quasi linear function, to form the output pulse. Weights g_y may all be one. Thus, a single burst of a neuron is able to express topological properties of a local connectivity.

Now we increase the threshold to a value near the sum of maximum input amplitudes, y). The neuron is now allowed to answer only in the case, a time function appears at the input, where each impulse has an equivalent, inherent delay. The neuron answers only this single code with an impulse. Since we know, that an average neuron has in average 7500 synapses, we can imagine the possible location sensitivity of an arrangement of a periphery laying generator neuron coupled with a central laying detector neuron. To verify the hypothesis, we used the simulator 'Neuronet' [4]. The experimental arrangement is as shown.

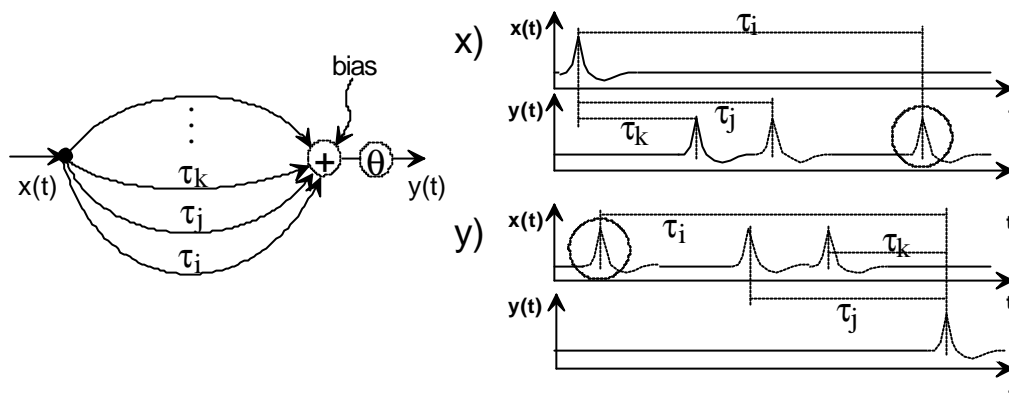


Fig. 23 Basic functions of a neuron or a neural group: code generation (burst submission) with low bias x) and code detection y) with high bias. Note, that the output events from x) {k,j,i} have been time-inverting to excite the same neuron, to be the input events in y) {i,j,k}

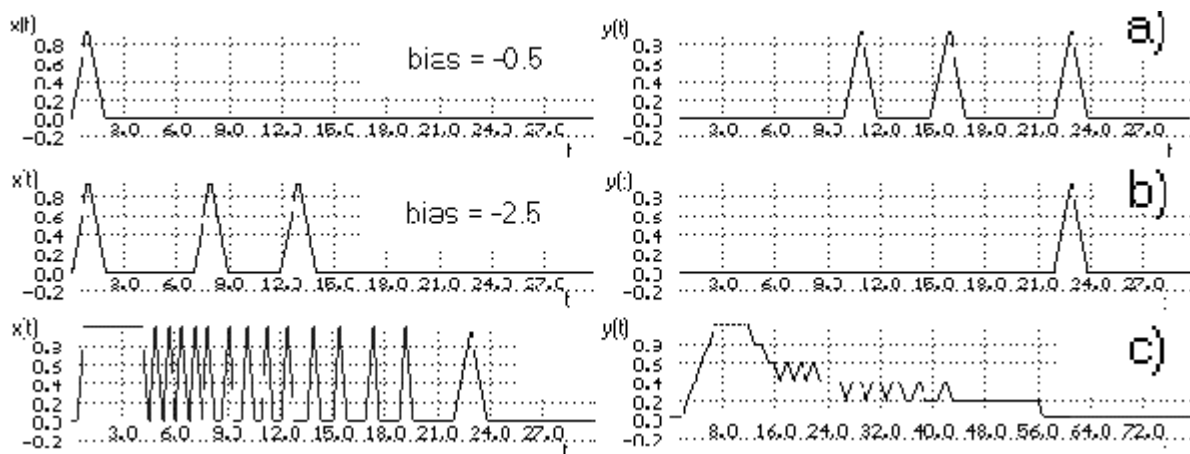


Fig. 24 Neuronet Simulation results for following Fig 25. Heinz/Puschmann/Schoel 1994. Input left, Output right. Code Generation a), code detection b), bias generation c)

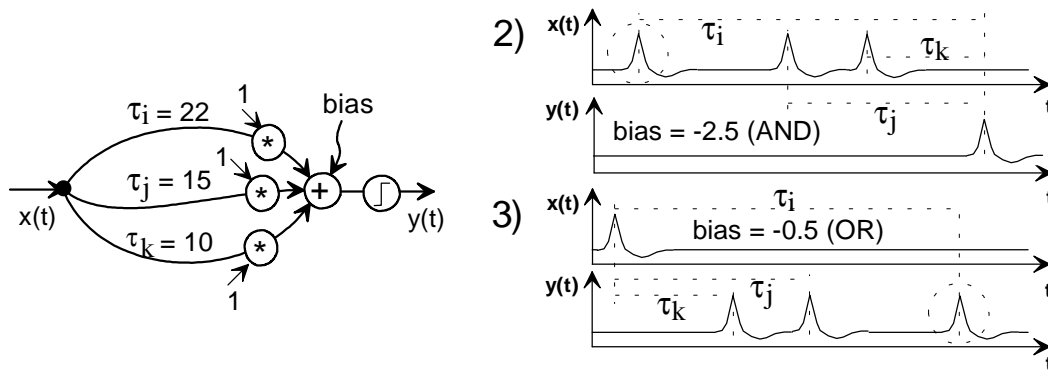


Fig. 25 Experimental arrangement: 2) Code detection, 3) Code generation.

Really we find, that the same neuron detects and generates code with an inverse timing. What would the Lord say to us? The nature has found a simple method to avoid neighbourhood excitement. If we have two equal designed neurons with unidirectional dendrites/axons with equal connections together, the *excitement of one neuron can never jump to the other!* Simply, the neural mass of equality in statistical connectivity decomposes to functional different units. Uni-directionality supposed, a neuron outputs an inverse code to the accepted input code.

18. Experiment: Addressing via Single Channel, Coding, Burst as Address

The main question in biological systems is, how to address different neurons in stochastic connected networks. To simplify a burst-submitting or -receiving circuit, we use a medical, neuro-anatomic drawing. The function of the equivalent circuit Fig. b) is as time function $y(t)$:

$$(24) \quad y(t) = \theta(x(t - \tau_1) + x(t - \tau_2) + x(t - \tau_3) + \dots + x(t - \tau_i) + bias)$$

Dependent of the *bias* the equation has two very different solutions:

- If the *bias* is near the possible maximum *max* sum of *i* inputs, *i max*, each term produces an impulse at the output. That means, a single impulse $x(t)$ produces a burst $y(t)$.
- If the *bias* is negative with $(i-1) max$, the condition for excitement is, that all partial waves must appear simultaneous at the adder. In other words that means, we need a definitive burst, including all inherent delays to excite the neuron one times.

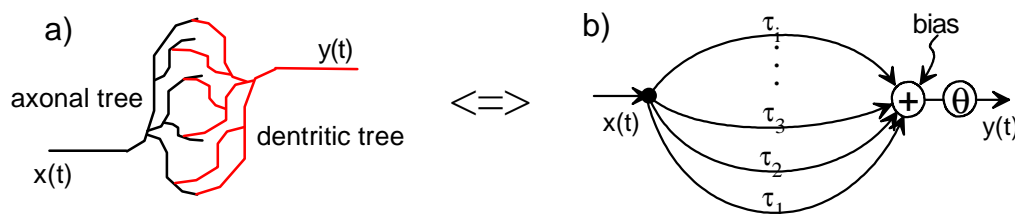


Fig. 26 Tree-like connectivity scheme of two neurons. Anatomical drawing a) and equivalent circuit b)

Now, lets couple different burst circuits together in pairs of senders and receivers. Notice, that different senders x_i can (statistically) pulse to the one transmission wire y without of dramatically malfunction. Each receiver will only register the burst, that codes 'its' sender, that means, a sender, using the mask m , where receiver uses the inverse m^* .

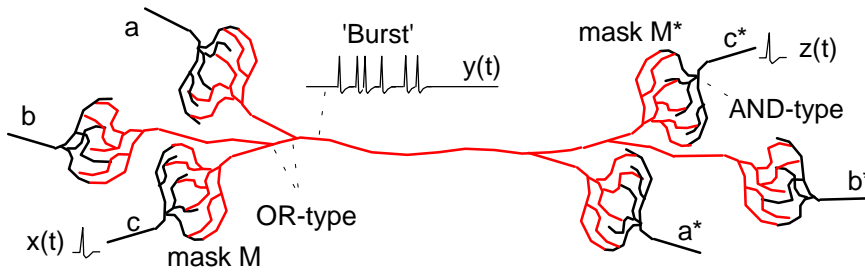


Fig. 27 Directed, multiple data transmission via one axon. Only that receiver is addressed, that owns the complementary key in form of the inverse mask M^* of the sender. Information flows directed $a \rightarrow a^*$, $b \rightarrow b^*$, $c \rightarrow c^*$

The last question may be, how to construct this inverse mask.

We remember, the criterion for interference are equal delays $\tau_j + \tau_j^*$ on all paths over the total distance between sender and receiver. The index ($*$) stands for the receiver side.

$$(25) \quad \forall_{j=1}^n \tau_j + \tau_j^* = const. = \tau$$

Thus, the sum of sender mask M and receiver mask M^* have to be equal in components:

$$(26) \quad M + M^* = T$$

where T is the unit vector multiplied with τ . The solution to construct an inverse mask M^* , receiving a burst with mask M is

$$(27) \quad M = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_i \end{pmatrix}; \quad T = \tau \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}; \quad M^* = \begin{pmatrix} \tau - \tau_1 \\ \tau - \tau_2 \\ \vdots \\ \tau - \tau_i \end{pmatrix}$$

Because interference systems are independent of absolute time, the total delay τ between sender and receiver location can have any positive value greater the sum of the greatest complement of masks (negative delays are forbidden).

This information selection can influence two mechanisms:

- Data addressing is possible only between a neuron with mask M and a neuron with the inverse mask M^* .
- Neighbourly excitement only is possible, if they have inverse masks (neighbourhood inhibition).

19. Experiment: Interference Systems with Different Speeds, Coloured Interference System

Supposed, we have a lot of parallel wires, each carrying impulses in parallel. Using a parametric description (GnuPlot) with normalised variables $x = X/X_0$, $t = T/T_0$ and $V = X_0/T_0$ it is possible to suppose initial conditions in the form

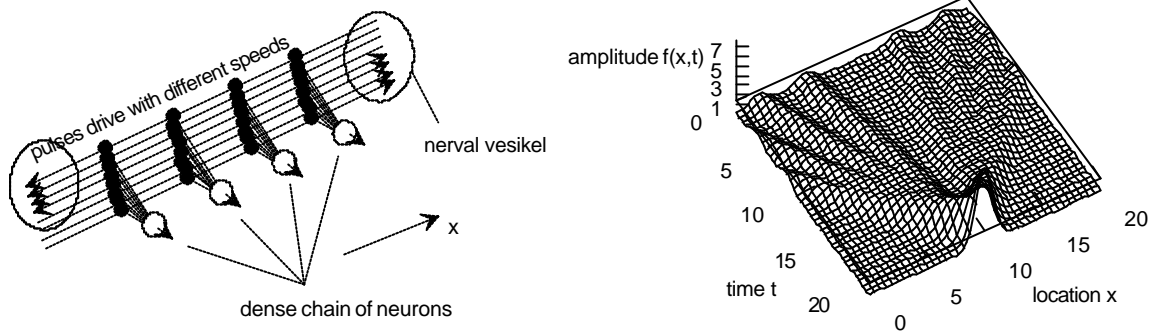


Fig. 28 Wave expansion. Different pulses with different partial speeds grows to interference in a linear array arrangement (GnuPlot drawing)

$$(27) \text{ splot } x, t, \text{ imp}(t-0.8*x+6) + \text{ imp}(t-0.6*x+2) + \text{ imp}(t-0.4*x-2) + \text{ imp}(t-0.2*x-6) + \text{ imp}(t+0.8*x-26) + \text{ imp}(t+0.6*x-22) + \text{ imp}(t+0.4*x-18) + \text{ imp}(t+0.2*x-14)$$

We suppose further, all parallel wires lay very closed together. They are packed with neurons having one input per wire. So we can find very discrete locations of interference, when ever a location of excitement has reasoned the impulses group. We call systems 'coloured', having different speeds on different wires or having some more carriers with different speeds into one wire. It is simple to see, that coloured interference systems have a small possibility to be re-constructable (for example by surgery), when ever they are destroyed.

20. Experiment: Gating, Bias Generation

Neural computation needs gating levels or potentials, for example for pausing an aktive task over some time. How to create week potentials for example for the moving- or zooming-control with pulses?

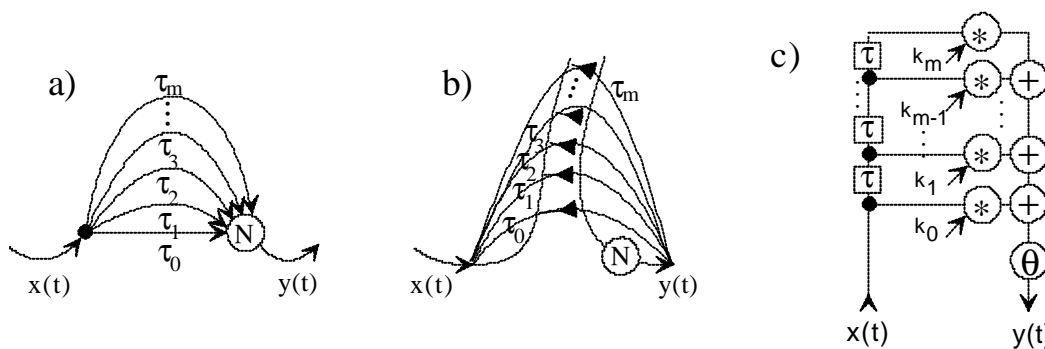


Fig. 29 Parallel (a) and serial (b) neural circuits to produce gating levels. An equivalent circuit gets the form of a Finite Impulse Response Filter c)

A parallel or serial connected delay chain a) or b) may have the delay properties

$$(28) \tau_{i+1} = \tau_i + \Delta\tau, \quad 0 \leq i < n.$$

To hold the output $y(t)$ constant, we can influence a lot of parameters: weights, input time function, delay distance etc..

$$(29) y(t) = \sum_{i=1}^m k_i x(t - i\Delta\tau) = const$$

An interesting case we find, where the sum of weights is smaller or equal a constant c near one

$$(30) \sum_{i=1}^m k_i \leq c.$$

For simulation we used all $\tau_i = 1$. The weights k_j are unique 0.2. The neuron has 6 inputs. a) input $x(t)$, b) output $y(t)$ with bias = 0 and linear threshold function θ . Find the Neuronet simulation results in Fig. 24c).

Using an incremental linear threshold function q now an incoming pulse series produces a gating level at the output. The incremental stage in the output function is as high as the weight of one input. As more inputs we use, as smaller are the weights, as smoother can be the output level.

21. Experiment: **Points of Interference, Code Locations**

The chapter 'over-conditioning' has shown some problems to create real projections over more then three channels. Now we will use 16 channels to create an interference projection from an detector field with a neural arrangement in form of 'GH' to another field. Connections between both fields are shown. Wiring delays are equal.

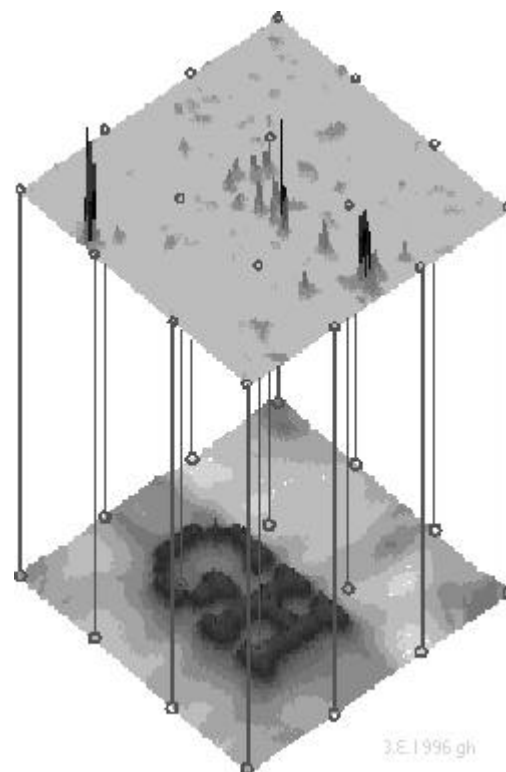


Fig. 30 Interference projection over 16 channels. The bottom GH-arrangement (reconstruction) relates to a three points excitment in the top projection

It seems important to see, that only a small amount of interference locations occure in the projection (top). May be, the top detector field consists of neurons. Then an occurence of the GH-arrangement with any reproducable timing leads to an excitement of the shown three neurons. Other arrangements suggest other configurations of exciting neurons. To reach the smallest amount of neurons it is necessary to use a large number of different delays or to adapt the different delays by a learning algorithm. The drawn projection into a homogenous detector field so is the worst case to get a high abstraction niveau for a single neuron.

Summary

We introduced time functions on wires carrying signals with low speed while we ask for interference conditions in 3D-networks. We find *locations of interference* combined with time functions and net structure as an universal addressing principle of information in stochastic connected networks. We find, the *function of one neuron is coded through its relative location*.

We construct the simplest neural interference circuit and find *mirrored projections* of excitement between a generator and a detector field. (Pictures show *non-mirrored reconstructions* of generator spaces done with inverse masks).

Changing the speed or changing geometrical properties of interference circuits vary the projective properties. Different *speeds* between generator and detector *zoom* the projected image. Different *delays on axonal wires move* the projection back and forth.

Addition and multiplication of time functions demonstrate the high sensitive possibilities of a neuron to detect an excitement. To find mathematical possibilities to reconstruct interference spaces, we introduce a *delay mask* to characterise the relation between location of excitement and channel data. The mask gives the chance, to interpret the contents of parallel channel data streams. We differentiate between *reconstruction of a generating space* and interferential *projection into a detecting space*. In dependence of the masks usage direction (positive or negative) we get a *mirrored projection* onto a neural detector or a *non-mirrored reconstruction* of the generator space. We find as the main difference between projection and reconstruction an *inverted time axis*.

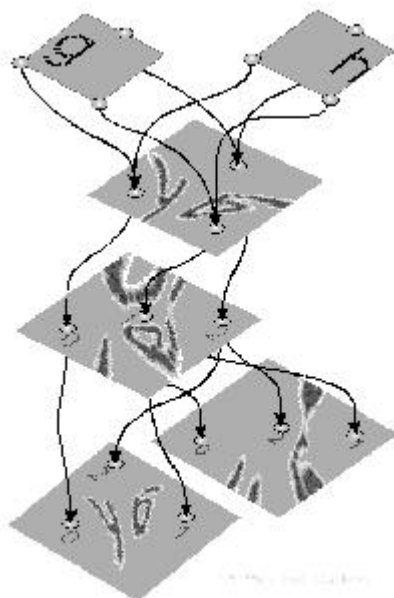


Fig. 31 Projection into different receiver fields. Variation of the channel source arrangement causes partial zooming- and moving effects, partial image limitation, distortions or multiple excitements, topology stays unchanged. g and h are not separable.

A closer look to projections shows *over-conditioning problems* comparable to problems in optical projecting systems. Projections become as more accurate, as more the different paths have equal delays.

Introducing the terms '*self- and cross-interference*' allows to research neural cross interference pattern. Using dense impulse series with wavelength being smaller the field size, superimposition shows cross interference mapping of peaks that are coded in form of pulse frequencies.

In cross interference we find the reason of *phantom excitement*. Heavy cross interference supposed, projections become absorbed within phantom excitement (pain).

The construction of interference circuits with more then one detector/generator space shows conjunctive projections (*dermatome projections*). Here it is not possible, to differ between the excitement sources.

A closer look to a single neuron shows the possibility of elementary, neural basic functions like *code generation* and *code detection*. We find a reason for *neighbourhood inhibition* between

closest coupled, near located neurons. We introduce a *colored interference system* having wires with different speeds. We find, that nerval colored wiring themselves defines possible locations of interference for a group of time functions.

We find *bursts* as the natural capability to transmit data with different destination addresses on the same axonal wire; as the **natural principle for addressed communication** in chaotic connected networks.

An inspection of closed synaptic arrangements shows possibilities to create *week gating potentials*, needed to control zooming or moving mechanisms.

The homepage [3] offers additional 30 MB of illustrations, movies and a download directory.

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